On the estimation of uncertainties for the accelerometer calibration using laser interferometry

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Abstract
In practice, traceability leads to improve the accuracy and agreement among the results obtained using different means of measurement. However, in every calibration within the traceability chain there is a certain increment of the uncertainty of measurement. The uncertainty of measurement gives an idea of the length of the traceability chain and the accuracy of the methods used to calibrate the measurement standard. The case when the linear acceleration quantity is measured using a laser interferometer, traceability to length through the calibration of the laser in wavelength, and time and frequency through the calibration of a sine generator and a frequency counter, is discussed. It highlights that an important portion of type B uncertainty related with these reference standards depends on two factors, their traceability chain to basic standards of length and frequency, and the measurement accuracy achieved within the traceability chain.

Keywords: traceability, uncertainty, type B uncertainty, reference standards, traceability chain.

1. Introduction

In rigid body mechanics, there are six dynamic quantities, i.e., linear displacement, linear velocity, linear acceleration, angle of rotation, angular velocity and, angular acceleration. They are often referred to as vibrations, the term vibrations is used, in general, to describe the repetitive motion of a particle relative to a stationary reference frame (frame of reference). INMAN [1996] states that the vibrational properties of engineering devices are often limiting factors in their performance. In other words, by measuring the dynamic performance of a specific device we can estimate values of its constitutive elements, i.e., damping, elasticity and inertia. The better accuracy in the measurement the better estimate of the constitutive elements and dynamic performance of the device which is in study.

With accurate measurements we can have a better understanding of the dynamic performance of a device, furthermore an accurate mathematical model leads us to predict some specific behavior under stated conditions. However, the measurement accuracy depends on the stability of the conditions during the realization of the measurement and the traceability of the standards. The international vocabulary of basic and general terms in metrology [1993] defines traceability as the property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national of international standards, through an unbroken chain of comparisons all having stated uncertainties.

In practice, traceability leads to improve the accuracy and agreement among the results obtained using different means of measurement. However, in every calibration within the traceability chain there is a certain increment of the uncertainty of measurement. The uncertainty of measurement gives an idea of the length of the traceability chain and the accuracy of the methods used to calibrate the measurement standard. The six dynamic quantities mentioned above are derived quantities in the International System of Units, SI, therefore their measurements are traceable to basic quantities. The principles of measurement used within the traceability chain define the basic quantities that act as fundamental references.

Nowadays, for that case when the linear acceleration quantity is measured using a laser interferometer, it is necessary to have traceability to length through the calibration of a laser in wavelength, and frequency through the calibration of a sine generator and a frequency counter. This fact is based on important scientific, experimental and technological improvements. Since the

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end of the 19th century, marked by Michelson's famous experimental achievements which led to the redefinition of the length standard [1903], the measurements of length have been shown an important improvement on the accuracy of measurement. From 1960 the use of laser, as a source of light, have been contributing to the development of several experimental methods using laser interferometers. Developments on instrumentation, digital computers and optical devices have been contributing to our current knowledge.

This paper shows that once the uncertainty of the laser interferometer is determined, it is used to the measurement of the linear acceleration quantity and to estimate the sensitivity of the accelerometer under calibration. In type A uncertainty are included some disturbing factors as, environmental conditions, repeatability, variability due to the method, variability due to the user and, variability due to the experimental procedure. They all can be considered as stochastic variables. On the other hand, an important portion of type B uncertainty related with the accelerometer calibration depends on two factors, their traceability chain to basic standards of length and frequency, and the measurement accuracy achieved within the traceability chain. The variables included in type B uncertainty are of deterministic type.

In this paper we will suppose that the motion of the measuring surface is equal to the motion of the accelerometer. Furthermore, it is considered that all the moving elements only have rigid body motion, this means that they work below their fundamental resonance frequency.

2. Michelson interferometer fundamentals

Albert Abraham Michelson was the first American citizen to win the Nobel Prize in Physics, this was in 1907. Michelson made several scientific contributions and led to redirect the course of further developments, most of them based on experimental results. In 1896, Michelson carried out the first measurement of the length of the Pt-Ir bar that was the international prototype of the meter in terms of the wavelength of the red cadmium radiation. While the idea of the wavelength of a monochromatic source as a natural standard of length had been suggested earlier by Babinet and Fizeau, it was Michelson's work which demonstrated its feasibility and led, in 1960, to the redefinition of the metre in terms of the wavelength of the orange radiation of ^{86}Kr.

Nowadays the He-Ne laser is widely used as a reference light source mainly because of its characteristics, i.e., monochromatic, collimated, spatially coherent and, temporally coherent. The typical wavelength of the He-Ne laser is 632,8 nm, approximately. The Michelson interferometer uses the amplitude division method, this method is described as two beams which are obtained from a single laser source by division of the amplitude over the same section of the wavefront, more information can be found in Hariharan [1985]. In this paper our main concern is dealing with an interferometer of homodyne, in other words, only one photodiode is used. The Michelson interferometer is shown in Figure 1.

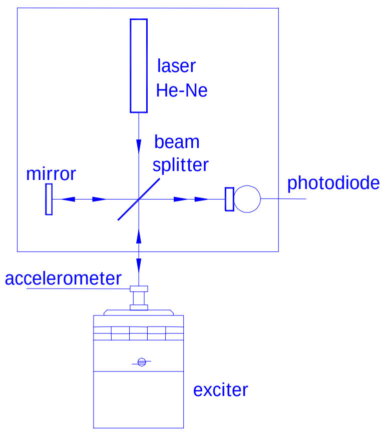

Figure 1. Michelson interferometer used to calibrate accelerometers

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2.1 Laser Doppler interferometry

Before analyzing the theory involved in the Michelson interferometer, it is necessary to consider the wave nature of light. Light is an electromagnetic phenomenon described by Maxwell [1954]. The theory considers two functions in vector space related to time and describing the electric and magnetic fields. However, in cases like this, it is enough to consider only the electric field. This treatment is used for the case of a limited plane wave and is suitable enough as an approach for the experimental purposes of this work, a better explanation is in [Pain, 1997]. Therefore, the laser beam is considered as an electric field which has an intensity, A, and is divided in two beams at the beam splitter. The reference beam is first reflected at the beam splitter and latter at the fixed mirror,

$$E_1 = A_1 \exp j(\omega t + \varphi_1) \tag{1}$$

where A_1 is the electric field intensity, ω is the reference circular frequency and, φ_1 is the optical phase. Besides, the measuring beam is first transmitted through the beam splitter and then reflected at the measuring surface, when the beam comes back from the measuring surface it can be expressed as,

$$E_2 = A_2 \exp j(\omega' t - \varphi_2) \tag{2}$$

where A_2 is the electric field intensity, $\omega' = \omega/(1 \pm v/c)$, is the sifted frequency by the Doppler effect, v is the instantaneous velocity of the measuring surface, c is the speed of light and, φ_2 is the optical phase of the measuring beam, in Haskell [1971] and Cloud [1995] is derived a development of laser Doppler interferometry. The two beams combine to each other at the beam splitter, then they are directed to the photodiode which measures the irradiance, $I = |E_1 + E_2|^2$. The irradiance of both the reference beam and the measuring beam is,

$$I = |E_1 + E_2|^2$$
$$I = 2A_1A_2 \exp j(\omega + \omega')t \cos(\varphi_1 + \varphi_2) + 2A_1A_2 \exp j(\omega - \omega')t \cos(\varphi_1 - \varphi_2) \tag{3}$$

The irradiance, I, depends on two terms: i. the phase, $\varphi = \varphi_1 \pm \varphi_2$, between beams E_1 and E_2 and; ii. the frequency term, $\exp j(\omega \pm \omega')t$. However, the first term in equation 3 represents an irradiance that oscillates at frequencies equal or greater than ω or ω', these frequencies are too large to be detected by the photodiode; only a time-average irradiance, which is essentially constant, would be produced by this term. On the other hand, the frequency of the last term is proportional to the difference between frequencies ω and ω'. If the measuring surface moves, the frequency of the last term is shifted due to the Doppler effect. The eq. 3 gives the complete expressions for the irradiance, but it cannot be completely measured, then the photodiode output can be expressed as,

$$I = 2A_1A_2 \exp j\omega_B t \cos \varphi \tag{4}$$

where, $\omega_B = \omega - \omega'$, is the so-called beat frequency and, $\varphi = [\delta + 2\xi(t)](2\pi/\lambda)$ is the optical phase; λ, is the He-Ne wavelength; δ, is the path difference between the two beams; $\xi(t) = \xi \sin(2\pi f t)$, is the simple harmonic motion (*shm*) described by the measuring surface, with an amplitude, ξ, and a constant frequency, f. It is important to keep in mind that: i. the optical phase, φ, depends on the displacement; ii. the frequency, ω_B, is proportional to the instantaneous velocity, both relative to the measuring surface and; iii. the frequencies, ω and ω', are close enough so that the difference frequency can be measured with ordinary photodiodes and, the output is a harmonic wave expressed as a function of time. When the measuring surface has a *shm*, then the photodiode output will vary harmonically too, this is seen in figure 2.

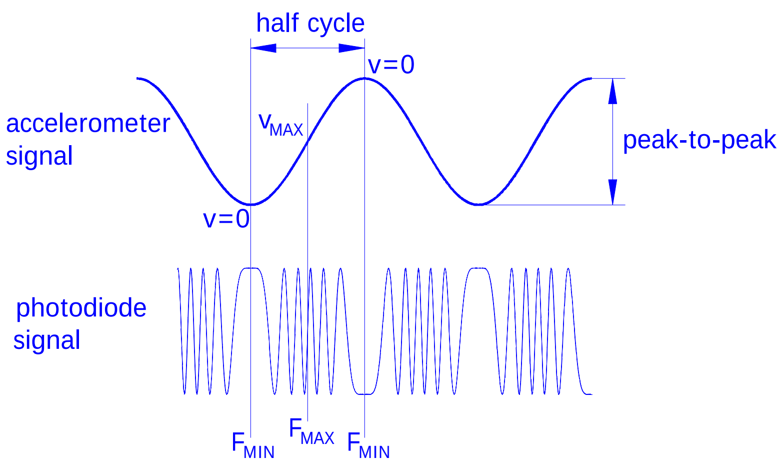

Figure 2. (a) simple harmonic motion of the measuring surface, (b) harmonic output of the photodiode.

2.2 Alignment

Observing the figure 2, we can notice that, at the photodiode output, every pulse has different frequencies because the accelerometer has a *shm* which has a different velocity at every point. Therefore, it is important to have only one fringe, at a time, on the photodiode; otherwise the frequency of the photodiode output would be higher, depending on the number of fringes on the photodiode and the instantaneous velocity of the accelerometer. Actually, the international standards, neither the ISO 5347-1:1993 nor the ISO 16063-1:1998 includes a topic about the alignment. This may lead to slight differences in the results because, if more than one fringe is on the photodiode, a higher frequency would be read and, it would be interpreted as a higher displacement, or higher acceleration level, which would conduct to obtain a lower accelerometer sensitivity.

For our application, regarding the alignment of the Michelson interferometer, we can imagine a monochromatic point source S located at a finite distance gives rise to two virtual point sources, S_1 and S_2, which are the images of S reflected in M_1 and M_2. Non-localized fringes are then observed on a screen placed at O. If, as shown in figure 3(a) the line S_1S_2 is parallel to AO, the fringes are circular; if, as in figure 3(b), S_1S_2 is at right angles to AO, they are parallel straight lines. In practice, the exciter which moves the accelerometer may produce tilting motion, for this case, an alignment similar to the shown in figure 3(b) may be less sensitive to tilting than the one shown in figure 3(a).

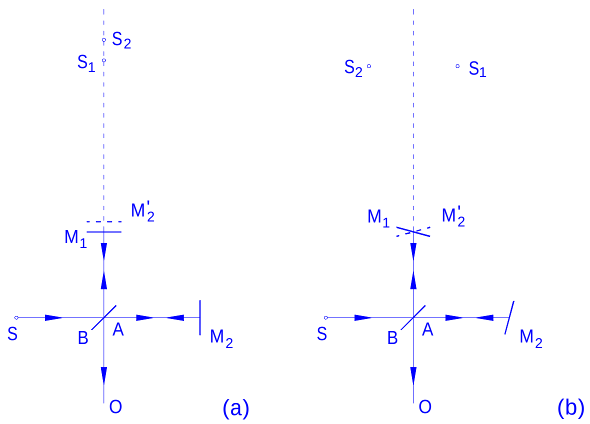

Figure 3. Formation of non-localized fringes in a Michelson interferometer, Hariharan[1985]

It is important to note that, when one fringe impinges the photodiode at a time, the rms output of the photodiode is not the highest. If we look for the highest rms level of the photodiode output, then we need about three fringes on its sensitive surface and, then a higher frequency would be read in its signal output. Additionally, other sources of noise may be found, i.e., i. if a metallic mirror is used, its roughness should be less than the laser wavelength, otherwise interference due to diffraction may appear; ii. it is advisable to use a one mode laser, otherwise more than one frequency may be present in the measurements; iii. the longer the laser tube the more sensitive to temperature changes, this may produce slight variations in frequency. Further discussion of this topic is not possible in this paper due to the lack of space, however, readers interested in this topics may find additional information in the references shown in section 6.

3. Frequency ratio method (fringe counting)

In this method two different frequencies are measured, they are related to the dynamic displacement, $\xi(t)$, which is exerted upon the accelerometer during its calibration. The first of these frequencies is the 'excitation frequency' on the vibration exciter, it comes from a sine generator. The moving element of the vibration exciter has a metallic plate with a polished surface, the accelerometer is bolted directly over the polished surface, this surface is called the 'measuring surface'. When a *shm* is shown by the measuring surface, the accelerometer output is referred to both the excitation frequency and the beat frequency measured with the interferometer.

When the measuring surface moves backward or forward a distance, $\lambda/2$, the fringes move from the dark side to the bright one. So, at a stated frequency and acceleration level, there will be a constant number of fringes, or optical pulses, during each mechanical vibration cycle, in other words, the fringe frequency equals the mechanical frequency times the number of fringes, as shown in figure 2. This number of fringes is proportional to both the displacement and acceleration levels.

It was stated in equation 4 that the frequency of the light irradiance , measured by the photodiode, is proportional to the forced *shm* described by the measuring surface. The light irradiance level, which indeed is an average value, varies harmonically too. Looking at equation 4, since the values of the cosine vary in the range from –1 to +1, the maximum light irradiance level is,

$$cos\{[\delta + 2\xi(t)](2\pi/\lambda)\} \approx \pm 1 \tag{5}$$

that is to say,

$$[\delta + 2\xi(t)](2\pi/\lambda) = 2\pi n \tag{6}$$

where, n, is an integer. Considering two consecutive values of the integer, i.e., n and $n+1$, when two consecutive maximums occur. Then, it is easy to find the relationship between the displacements, ξ_n and ξ_{n+1}, respectively. The above means that two consecutive maximums of light irradiance level are produced when the traveled distance of the measuring surface follows the next relationships,

$$[\delta + 2\xi(t)](2\pi/\lambda) = 2\pi n \tag{7}$$

and,

$$[\delta + 2\xi(t)](2\pi/\lambda) = 2\pi(n+1) \tag{8}$$

From the two equations 7 and 8, it can be seen that the traveled distance of the measuring surface to produce two consecutive light irradiance maxima is, $(\xi_{n+1} - \xi_n) = \lambda/2$. When the forced *shm* at the measuring surface has both constant frequency and constant displacement amplitude, the number of fringes of light irradiance maxima is constant too, as shown in figure 2.

In order to determine the number of fringes, it is necessary to calculate the dynamic displacement amplitude, $\xi(t)$, of the measuring surface. ISO 5347-1:1993 and ISO 16063-1:1998 use the following relation,

$$R_F = \frac{\xi \times 2(forward - bacward) \times 2(peak - to - peak) \times 2(complete\ cycle)}{\lambda} = \frac{\xi 8}{\lambda}$$

and we know that,

$$R_F = \frac{Fringe\ \ frequency}{shm\ \ frequency} \tag{9}$$

that is to say,

$$\xi = \lambda \cdot R_F / 8 \tag{10}$$

where, R_F, is called the frequency ratio which is the average number of fringes in several cycles of the *shm* described by the measuring surface. It is important to remember that for a He-Ne laser, the average wavelength, λ, is 632,8 *nm*, approximately. In the next section it is briefly shown the uncertainty budget for the estimation of the accelerometer sensitivity which is under calibration.

4. Uncertainty budget

4.1 Modeling the measurement, GUM [1995]

In most cases a measurand Y is not measured directly, buy is determined from N other quantities, X_1, X_2, . . . , X_N through a functional relationship f:

$$Y = f\left(X_1, X_2, \ldots, X_N\right) \tag{11}$$

However, an estimate of the measurand Y, denoted by y, is obtained from (11) using *input estimates* x_1, x_2, . . . , x_N for the values of the N quantities X_1, X_2, . . . , X_N. Thus the output estimate y, which is the result of the measurement, is given by

$$y = f\left(x_1, x_2, \ldots, x_N\right) \tag{12}$$

Each input estimate x_i and its associated standard uncertainty $u(x_i)$ are obtained from a distribution of possible values of the input quantity X_i. This probability distribution may be frequency based, that is, based on a series of observations $X_{i,k}$ of X_i, or it may be an a priori distribution. The Guide to the expression of uncertainty in measurement, *GUM [1995]*, states that Type A evaluations of standard uncertainty components are founded on frequency distributions while Type B evaluations are founded on a priori distributions. The standard uncertainty of y, where y is the estimate of the measurand Y and thus the result of the measurement, is obtained by appropriately combining the standard uncertainties of the input quantities x_1, x_2, . . . , x_N. This combined standard uncertainty of the estimate y is denoted by $u_c(y)$.

The combined standard uncertainty $u_c(y)$ is an estimated standard deviation and characterizes the dispersion of the values that could reasonably be attributed to the measurand. Equation (13) is based on a first-order Taylor series approximation of Y, and expresses what is termed in the GUM [1995], the *law of propagation of uncertainty*.

$$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i)u(x_j)r(x_i, x_j) \tag{13}$$

where $r(x_i,x_j)=r(x_j,x_i)$, and $-1\leq r(x_i,x_j)\leq +1$, is the *correlation coefficient* which characterizes the degree of correlation between x_i and x_j. Furthermore, although $u_c(y)$ can be universally used to express the uncertainty of a measurement result, in some applications it is necessary to give a measure of uncertainty that defines an interval about the measurement result that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand. The additional measure of uncertainty that meets the requirement of providing an interval of the kind indicated above is termed *expanded uncertainty* and is denoted by U. The U is obtained by multiplying the combined standard uncertainty $u_c(y)$ by a *coverage factor* k:

$$U = k \cdot u_c(y) \tag{14}$$

Then, the U can be interpreted as defining an interval about the measurement result that encompasses a large fraction p of the probability distribution characterized by that result and its combined standard uncertainty, and p is the *level of confidence* of the interval. To obtain the value of the coverage factor k that produces an interval corresponding to a specified level of confidence p, a t-distribution of the measurand y may be considered and the effective degrees of freedom ν_{eff} can be obtained from the Welch-Satterthwaite formula,

$$\nu_{eff} = \frac{u_c^4(y)}{\sum_{i=1}^{N} \frac{u_i^4(y)}{\nu_i}} \quad (15)$$

4.2 Sensitivity calibration results

The ISO 16063-1:1998 defines sensitivity as, for linear transducer, the ratio of the output to input during sinusoidal excitation parallel to a specified axis of sensitivity at mounting surface. The sensitivity is a complex quantity that varies with frequency. In general, the term sensitivity can be assumed as the relation of as output divided by an input, input and output can be different physical quantities. As mentioned above, piezoelectric accelerometers are among the most developed transducers, two types of accelerometer sensitivities are used: i. the charge sensitivity which has units of pC/(m/s^2) and, ii. the voltage sensitivity which has units of mV/(m/s^2). So far, several authors have been reporting that the charge sensitivity shows better stability than the voltage sensitivity, therefore in this section the uncertainty budget for the charge sensitivity is developed.

For the case when a Michelson interferometer is used, the instantaneous displacement of the measuring surface, ξ, can be obtained from equations (9) and (10)

$$\xi = \frac{\lambda\, R_F}{8} = \frac{\lambda\, F_F}{8\, F_E} \quad (16)$$

where F_F is the photodiode output frequency in *Hz*; F_E is the excitation frequency on the measuring surface in *Hz* and; λ is the wavelength of the He-Ne laser which is 632,8 *nm*, approximately. Since a shm is described by the measuring surface, it is possible to know the acceleration level, *a*,

$$a = (2\pi F_E)^2 \xi = \frac{\pi^2 \lambda\, F_F\ F_E}{2} \quad (17)$$

As mentioned above, the charge sensitivity of the accelerometer, S_C , is expressed in terms of the output electric charge, *q*, divided by the acceleration level, *a*, exerted on the measuring surface. Then, the charge sensitivity is expressed as,

$$S_C = q / a \quad (18)$$

However, the accelerometer is connected in series to a charge amplifier, this is necessary in order to convert the high impedance at the output of the accelerometer to a low impedance at the output of the charge amplifier. In this way, the output signal of the set, accelerometer plus charge amplifier, has a low impedance output and is suitable for connection to relatively low impedance input of measuring and analyzing instrumentation. On the other hand, the charge amplifier sensitivity, A_C, can be expressed as,

$$A_C = \frac{q}{E} \quad (19)$$

Substituting equations (17) and (19) into equation (18), it is possible to express the charge sensitivity of the accelerometer in terms of quantities which affect the calibration process.

$$S_C = \frac{2\,E}{\pi^2\,\lambda F_F\ F_E\ A_C} \quad (20)$$

Where,
S_C, charge sensitivity of the accelerometer, pC/(m/s^2)
E, charge amplifier output voltage, mV
λ, He-Ne laser wavelength, nm
F_F, photodiode output frequency, Hz
F_E, excitation frequency on the measuring surface, Hz
A_C, charge amplifier sensitivity, mV/pC

The law of propagation of uncertainties, equation (13), applied to the functional relationship of the charge amplifier sensitivity of the accelerometer gives,

$$u_c^2(S_C)=\left(\frac{\partial S_C}{\partial E}\right)^2 u^2(E)+\left(\frac{\partial S_C}{\partial \lambda}\right)^2 u^2(\lambda)+\left(\frac{\partial S_C}{\partial F_F}\right)^2 u^2(F_F)+\left(\frac{\partial S_C}{\partial F_E}\right)^2 u^2(F_E)+$$
$$\left(\frac{\partial S_C}{\partial A_C}\right)^2 u^2(A_C)+\frac{1}{2}\frac{\partial S_C}{\partial E}\frac{\partial S_C}{\partial F_F}u(E)u(F_F)r(E,F_F)+ \qquad (21)$$
$$\frac{1}{2}\frac{\partial S_C}{\partial E}\frac{\partial S_C}{\partial F_E}u(E)u(F_E)r(E,F_E)+\frac{1}{2}\frac{\partial S_C}{\partial F_F}\frac{\partial S_C}{\partial F_E}u(F_F)u(F_E)r(F_F,F_E)$$

The equation (21), which is based on a first-order Taylor series approximation, is applied to the measurement of an accelerometer. The calibration was made exciting the accelerometer with a *shm* at a frequency of 159,155 *Hz* and with an acceleration level of 40 m/s^2 approximately. The nominal sensitivity of the accelerometer is 0,99 $pC/(m/s^2)$. The results obtained from the uncertainty budget are shown in Table 1, at the end of this paper.
In order to check if some nonlinearity of S_C was significant, first-order and second-order terms in the Taylor series expansion were included in the law of propagation of uncertainties, the equivalent to equation (13). The result obtained for the combined uncertainty squared, $u_c^2(S_C)=1{,}23\times10^{-8}\left[pC/\left(m/s^2\right)\right]^2$, was practically the same for the simpler approximation. This result proves that the model for S_C does not have any significant nonlinearity.

5. Conclusions

The Michelson interferometer analyzed in section 2 has a moving mirror (for our purpose, it is the measuring surface) which describes a simple harmonic motion. The irradiance is measured by a fast response photodiode. The irradiance has two important components, the optical phase and the beat frequency. The optical phase depends on the displacement of the measuring surface and the beat frequency depends on the velocity of the measuring surface. The later is an important factor for measurements of phase.
The uncertainty budget, shown in table 1, indicates that the beat frequency obtained from the photodiode output is the main source of uncertainty. Therefore, further improvements to the experimental procedure are needed, i.e., the alignment of the interferometer (a proposal is included in section 2.2), the quality of the optical devices, the use of one-mode polarized laser, among the most important. On the other hand, changes in temperature less than 2 °C have small influence on the uncertainty, however, it is advisable to avoid air turbulence.
The second source of uncertainty is the uncertainty on the calibration of the charge amplifier. The charge amplifier is connected in series to the accelerometer, it is necessary in order to have a low impedance output, a brief explanation is included in section 4.2. It is recommended to have an extra-care during the electric calibration of the charge amplifier.
A correlation between the output voltage of the charge amplifier and the beat frequency of the photodiode output is identified, see table 1. The three correlation factor identified in the uncertainty budget have a negative sign, it is so because they can be thought as systematic variations and the uncertainty only takes into account random variations. In general, the correlation identification leads to reduce the uncertainty, the variability of stochastic factors increase the uncertainty while deterministic factors reduce it.

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Table 1. Uncertainty results of the charge sensitivity calibration

Quantity	Level (x_i)	Type A	Type B	$u^2(x_i)$	$\left(\frac{\partial S_C}{\partial x_i}\right)^2$	Factor^2	Contribution
Voltage, E	2 803,238 *mV*	0,007 mV	u_{TRAC}=0,028 mV u_{RESOL}=0,006 mV	0,000 9 mV^2	6,28x10^{-8}	5,44x10^{-11}	0,4 %
Wavelength, λ	632,8 *nm*	-	u_{TRAC}=3,3x10^{-14}m	1,09x10^{-27} m^2	1,23x10^{12}	1,34x10^{-15}	0,0 %
Beat frequency, F_F	80 498,929 *Hz*	10,042 Hz	u_{RESOL}=5,8x10^{-4} Hz	100,851 Hz^2	7,61x10^{11}	7,67x10^{-9}	60,9 %
Excitation frequency, F_E	159,155 *Hz*	2,214x10^{-5} Hz	u_{RESOL}=5,8x10^{-6} Hz	5,233x10^{-10} Hz^2	1,95x10^{-5}	1,02x10^{-14}	0,0 %
Charge amplifier sensitivity, A_C	99,7752 *mV/pC*	-	u_{TRAC}=0,01 mV/pC	0,000 1 mV^2/pC^2	4,95x10^{-5}	4,95x10^{-9}	39,3 %

Correlation	1/2	$\frac{\partial S_C}{\partial x_i}$	$\frac{\partial S_C}{\partial x_i}$	$u(x_i)$	$u(x_j)$	$r(x_i,x_j)$		
E - F_F	0,5	2,51x10^{-4}	-8,72x10^{-6}	0,029 5 mV	10,042 4 Hz	0,26	-8,40x10^{-11}	-0,7 %
$E - F_E$	0,5	2,51x10^{-4}	-4,41x10^{-3}	0,029 5 mV	0,000 02 Hz	0,06	-2,23x10^{-14}	0,0 %
$F_F - F_E$	0,5	-8,72x10^{-6}	-4,41x10^{-3}	10,042 4 Hz	0,000 02 Hz	-0,62	-2,74x10^{-12}	0,0 %

Combined uncertainty squared	$u_c^2(S_C)$	1,26x10^{-8}	[pC/(m/s^2)]^2
Combined uncertainty	$u_c(S_C)$	0,000 11	pC/(m/s^2)
Effective degrees of freedom	ν_{eff}	**6**	
Coverage factor	k	**2,65**	
Level of confidence	p	**≈ 95**	%
Expanded uncertainty in pC/(m/s^2)	U	**0,000 29**	pC/(m/s^2)
Expanded uncertainty in %	U	**0,03**	%
Charge sensitivity of the accelerometer	S_C	**0,9931**	pC/(m/s^2)